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FISHER'S FORMULA FOR INDEX NUMBERS

WARREN M. PERSONS

PROFESSOR IRVING FISHER, in a paper read at the last annual meeting of the American Statistical Association,[1] proposed a formula for the computation of index numbers of prices and of quantities which, he maintained, was "the best for all purposes." [2] The formula which he recommended for indices of prices and quantities as "ideal" is the geometric mean between two ratios of aggregates. The aggregates depend upon a combination of the prices and quantities of commodities in a given year with those in a base year.[3] The data necessary for the computation of Fisher's formula for a series of years consist of prices and quantities in every year of all the commodities selected for the index. For an *exact* comparison of average prices or quantities in two years, by Fisher's method, it is not sufficient to compute indices for those two years relative to a third year as a fixed base; it is necessary to use one of the two years compared as the base. Extensive data and laborious computations are required for the construction of Fisher's indices.

Professor Fisher has designated several types of averages as biased and hence unreliable. One of the averages he condemns, because it will have a downward bias, is the "geometric with weights dependent on prices at or near the base year." [4] Since this type of average was used by Professor E. E. Day in computing

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[1] See the *Quarterly Publication of the American Statistical Association*, March, 1921, pp. 533–551.

[2] *Ibid.*, p. 546. Mr. C. M. Walsh also recommends Fisher's index number as the "best" (*ibid.*, pp. 543–544).

[3] Professor Fisher's formulas for indices of prices and quantities of a given year are, respectively:

$$P_n = \sqrt{\frac{\Sigma\ p_nq_n}{\Sigma\ p_oq_n} \cdot \frac{\Sigma\ p_nq_o}{\Sigma\ p_oq_o}}$$

$$Q_n = \sqrt{\frac{\Sigma\ p_nq_n}{\Sigma\ p_nq_o} \cdot \frac{\Sigma\ p_oq_n}{\Sigma\ p_oq_o}}$$

where Σ = "the sum of such terms as"
p_n = the price of any commodity in a given year (or period)
q_n = the quantity of that commodity in the given year
p_o = the price of that commodity in the base year
q_o = the quantity of that commodity in the base year.

Fisher's index number of quantities, as is evident from the formula, is obtainable from his index number of prices by interchanging the p's and q's. Any statement concerning prices made for one of the two index numbers may, therefore, be matched by a similar statement concerning quantities for the other index.

Fisher's index number of prices for any year is the geometric mean of two ratios. Each of the two ratios is obtained by using as antecedent and consequent a pair of aggregates expressed in dollars (or some other monetary unit). The first of the two ratios has as its antecedent (1) the aggregate value of all the commodities considered at quantities and *prices in the given year* and as its consequent (2) the aggregate value of the same commodities and same quantities computed at the *prices in the base year*. The second of the two ratios has as its antecedent (1) the aggregate value of all the commodities at *prices in the given year* but quantities in the base year and as its consequent (2) the aggregate value of the same commodities at quantities and *prices in the base year*. *Each* of the two ratios contains prices of the given year in the antecedent and prices of the base year in the consequent.

Fisher's index number of quantities for any year is, likewise, the geometric mean of two ratios. Each of the two ratios is obtained by using as antecedent and consequent a pair of aggregates expressed in dollars (or some other monetary unit). The first of the two ratios has as its antecedent (1) the aggregate value of all the commodities considered at prices and *quantities in the given year* and as its consequent (2) the aggregate value of the same commodities and same prices computed by using the *quantities in the base year*. The second of the two ratios has as its antecedent (1) the aggregate value of all the commodities at *quantities in the given year* but prices in the base year and as its consequent (2) the aggregate value of the same commodities at prices and *quantities of the base year*. *Each* of the two ratios contains quantities of the given year in the antecedent and quantities of the base year in the consequent.

These formulas for the index numbers, P_n and Q_n, may be expressed in terms of relative prices, $\frac{p_n}{p_o}$, $\left(\text{or relative quantities, } \frac{q_n}{q_o}\right)$ instead of actual prices (p_n) or actual quantities (q_n), thus

$$P_n = \sqrt{\frac{\Sigma\ p_nq_n}{\Sigma\left(p_nq_n \cdot \frac{p_o}{p_n}\right)} \cdot \frac{\Sigma\left(p_oq_o \cdot \frac{p_n}{p_o}\right)}{\Sigma\ p_oq_o}}$$

$$= \sqrt{\begin{pmatrix}\text{Harmonic mean of price}\\ \text{relatives weighted by}\\ \text{values in the given year}\end{pmatrix} \cdot \begin{pmatrix}\text{Arithmetic mean of price}\\ \text{relatives weighted by}\\ \text{values in the base year}\end{pmatrix}}$$

and

$$Q_n = \sqrt{\frac{\Sigma\ p_nq_n}{\Sigma\left(p_nq_n \cdot \frac{q_o}{q_n}\right)} \cdot \frac{\Sigma\left(p_oq_o \cdot \frac{q_n}{q_o}\right)}{\Sigma\ p_oq_o}}$$

$$= \sqrt{\begin{pmatrix}\text{Harmonic mean of quantity}\\ \text{relatives weighted}\\ \text{by values in the given year}\end{pmatrix} \cdot \begin{pmatrix}\text{Arithmetic mean of quantity}\\ \text{relatives weighted}\\ \text{by values in the base year}\end{pmatrix}};$$

also,

$$Q_n = \sqrt{\frac{\dfrac{\Sigma\ p_nq_n}{\Sigma\ p_oq_o}}{\dfrac{\Sigma\left(p_oq_o \cdot \frac{p_n}{p_o}\right)}{\Sigma\ p_oq_o}} \cdot \frac{\dfrac{\Sigma\left(p_nq_n \cdot \frac{p_o}{p_n}\right)}{\Sigma\ p_nq_n}}{\dfrac{\Sigma\ p_oq_o}{\Sigma\ p_nq_n}}}$$

$$= \frac{V_n}{V_o} \cdot \sqrt{\frac{1}{\begin{pmatrix}\text{Arithmetic mean of price}\\ \text{relatives weighted by}\\ \text{values in the base year}\end{pmatrix}} \cdot \frac{1}{\begin{pmatrix}\text{Harmonic mean of price}\\ \text{relatives weighted by}\\ \text{values in the given year}\end{pmatrix}}}$$

and therefore,

$$P_nQ_n = \frac{V_n}{V_o}\ .$$

where
V_n = aggregate values of all commodities in the given year
V_o = aggregate values of all commodities in the base year.

From the formulas given above it is clear that Fisher's index numbers for both prices and quantities have *variable weights* from year to year. In both indices there are two sets of weights; the weights assigned to the several relatives in the two sets are proportional, respectively, (1) to the values of those commodities in the given year and (2) to the values of these commodities in the base year. As the given year changes the first set of weights also changes.

[4] *Ibid.*, p. 536. Professor Fisher intimates in his paper (and he has stated to the present writer) that the simple geometric mean is preferable to the geometric mean weighted by values "at or near the base year." The simple geometric means for prices and quantities of twelve leading crops relative to the base 1910 are as follows for the period 1880–1920:

	1880	1885	1890	1895	1900	1905	1910	1915	1920*
Prices	100.8	83.9	94.5	62.0	77.7	81.6	100.0	102.8	154.0
Quantities...	41.9	49.6	49.2	65.0	73.5	92.2	100.0	119.1	131.0

* Sugar omitted in 1920.

The simple geometric means of prices differ from the weighted geometric means and from Fisher's indices (see Table II, p. 106 of this article) by as much as 11 per cent and the simple geometric means of quantities differ from the weighted geometric means and from Fisher's indices by as much as 18 per cent. Unweighted geometric means, contrary to Fisher's opinion, are much less reliable than weighted means.

indices of the physical volume of production for agriculture, mining, and manufacture and since the Harvard University Committee on Economic Research has work in progress which will require the computation of index numbers of both prices and quantities a critical examination of Fisher's index number is undertaken in the present study. Is it more reliable than the geometric average with constant weights? If it is more reliable does the added accuracy justify the greatly increased work involved, at least double or treble that required by the geometric average with constant weights?[5]

Professor Edmund E. Day, as we have said, has recently used in the construction of index numbers of the physical volume of production for agriculture, mining, and manufacture, the geometric mean of relative quantities produced with constant weights; each commodity was weighted in every year according to its proportionate value in the base period 1909–13.[6] The object of the present paper is to compare (*a*) the results obtained by applying Fisher's formula to certain data utilized by Day with (*b*) the indices of quantities found by Day, and geometric indices of prices computed from the same data. The data that we shall use in making this comparison are the annual production in physical units (quantities) of twelve leading crops in the United States, and the average farm prices of these crops on December 1 of each year. The figures for the quantities produced of the twelve crops, 1879–1920, are given in the REVIEW for September, 1920, p. 253. The farm prices on December 1 of those crops, quinquennially 1880–1920 and annually 1910–20, the average annual value of each crop for the period 1909–13, the percentage distribution of values in 1909–13 (used as the constant weights in Day's indices of physical production for agriculture), and the range and mode of the distribution of values for 1879–1920, are given in Table I. Indices have been computed first, quinquennially for the period 1880–1920 inclusive, and second, annually for the years 1910–19. A slight revision of Day's indices as originally published has been necessary in order that precisely the same data might be used for both Fisher's indices and the geometric average with constant weights. Professor Day, quite appropriately for his purpose, included the value of cottonseed in getting the weight for cotton. In the present study his weights have been altered, excluding the value of cottonseed, and revised index numbers have been computed. The differences between the original and revised geometric averages with constant weights are 1.1 per cent or less for quantities and 2.2 per cent or less for prices.[7] These data and periods were selected for comparison of the Fisher and Day indices because (1) quantities produced are available for each year, (2) the data cover an extensive period of wide price changes, (3) the number of commodities is small, (4) in each year the values (and hence the weights) of certain commodities included, such as corn and cotton, are much larger than the values of other commodities, such as rice and flaxseed, giving a decided variation in weights among commodities, and (5) the percentage distribution of values (and hence the weights) fluctuates considerably during the period, giving a variation in weights from year to year. All of these features of the data should put the two index numbers to severe test with respect to each other, and any discrepancies between the two indices that appear should set an upper, rather than an average or lower

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[5] The relative amount of time required (given the data) by the computing staff to compute Fisher's indices and the geometric average with constant weights was ascertained by a time study. The results are as follows:

Index	Relative time required
Geometric average, constant weights	27
Fisher's method:	
Fixed base	51
Chain	100

These figures indicate that the geometric average with constant weights can be computed in about one-half the time necessary to secure Fisher's fixed base index, and in approximately one-fourth the time necessary to obtain Fisher's chain index.

In making these tests, the weighted geometric mean was calculated by looking up in a table of logarithms the figures to be averaged, multiplying these logarithms by the proper weights (using a slide rule), summing these products on the adding machine, dividing the sums of the products by the sum of the weights, looking up the antilogarithms of the quotients, and finally reducing these antilogarithms to relative numbers with 1910 as base.

Fisher's fixed base index was obtained by securing the products, $\Sigma p_o q_o$, etc., from the slide rule, summing the products on the adding machine, and then computing the final indices (based on four aggregates) by the use of logarithms.

In computing Fisher's chain index, the operation of chaining may introduce cumulative errors in the final result; hence, greater exactness is required in the earlier steps of the process. The slide rule would not have given a sufficient degree of accuracy for the products $\Sigma p_o q_o$, etc.; for that reason, the Monroe calculating machine was employed to perform these multiplications. From the products, the link and chain indices were secured by the use of logarithms.

We may add that Mr. Edwin Frickey has devised a special slide rule for computing geometric averages with constant weights which is being used by the computing section of the Committee on Economic Research. If this special rule be used the relative time required for computing the geometric average is reduced from 27 to 19.

[6] The formulas for the index numbers of prices and quantities (P'_n and Q'_n), using the geometric mean with constant weights, are, respectively,

$$P'_n = \sqrt[\Sigma v]{\left(\frac{p'_n}{p'_o}\right)^{v'} \left(\frac{p''_n}{p''_o}\right)^{v''} \cdots \left(\frac{p_n^{(r)}}{p_o^{(r)}}\right)^{v^{(r)}}}$$

$$Q'_n = \sqrt[\Sigma v]{\left(\frac{q'_n}{q'_o}\right)^{v'} \left(\frac{q''_n}{q''_o}\right)^{v''} \cdots \left(\frac{q_n^{(r)}}{q_o^{(r)}}\right)^{v^{(r)}}}$$

where

$p'_n, p''_n, \ldots p_n^{(r)}$ = prices of *r* commodities in the given year
$p'_o, p''_o, \ldots p_o^{(r)}$ = prices of those commodities in the base period
$q'_n, q''_n, \ldots q_n^{(r)}$ = quantities of those commodities in the given year
$q'_o, q''_o, \ldots q_o^{(r)}$ = quantities of those commodities in the base period
$v', v'', \ldots v_o^{(r)}$ = values of those commodities in the period used in determining the weights (which may or may not be the base period or year).

[7] The differences between Day's original and revised indices are as follows:

Year	DIFFERENCES		Year	DIFFERENCES	
	Quantity indices	Price indices		Quantity indices	Price indices
1880	+0.1	−0.7	1910	0	0
1885	−0.2	−0.3	1911	+1.0	−1.8
1890	+0.4	−0.9	1912	+0.2	−0.1
1895	−0.3	−0.2	1913	+0.5	−0.7
1900	+0.1	−0.2	1914	+0.7	−2.1
1905	−0.1	−0.2	1915	−0.6	−0.6
1910	0	0	1916	−0.1	−0.2
1915	−0.6	−0.6	1917	−0.4	−0.2
1920	−0.3	−1.3	1918	−0.1	−0.4
....	....	....	1919	−0.4	+0.5

TABLE I.—THE AVERAGE FARM PRICES ON DECEMBER 1 OF TWELVE LEADING CROPS, QUINQUENNIALLY 1880–1920 AND ANNUALLY 1910–20; THE AVERAGE VALUE OF EACH CROP FOR THE PERIOD 1909–13; AND THE PERCENTAGE DISTRIBUTION OF VALUES *

Year	Corn *cents per bushel*	Cotton *cents per pound*	Hay *dollars per ton*	Wheat *cents per bushel*	Oats *cents per bushel*	White potatoes *cents per bushel*	Sugar *cents per pound*	Barley *cents per bushel*	Tobacco *cents per pound*	Flaxseed *cents per bushel*	Rye *cents per bushel*	Rice *cents per bushel*
1880	39.6	9.8	11.65	95.1	36.0	48.3	7.97	66.6	8.2	107.9	75.6	132.9
1885	32.8	8.4	8.71	77.1	28.5	44.7	6.18	56.3	7.7	104.3	57.9	102.2
1890	50.0	8.6	8.18	83.3	41.6	75.3	5.19	62.6	8.3	106.1	62.6	102.2
1895	25.0	7.6	9.46	50.3	19.4	26.2	3.50	32.0	7.2	82.0	42.2	84.5
1900	35.1	9.2	9.72	62.0	25.4	42.3	4.44	40.5	6.6	144.4	49.8	87.7
1905	40.7	10.8	8.59	74.6	28.8	61.1	3.56	39.4	8.5	84.4	60.4	95.2
1910	48.0	14.1	12.14	88.3	34.4	55.7	3.93	57.8	9.3	231.7	71.5	67.8
1911	61.8	8.8	14.29	87.4	45.0	79.9	4.94	86.9	9.4	182.1	83.2	79.7
1912	48.7	11.9	11.79	76.0	31.9	50.5	4.05	50.5	10.8	114.7	66.3	93.5
1913	69.1	12.2	12.43	79.9	39.2	68.7	3.54	53.7	12.8	119.9	63.4	85.8
1914	64.4	6.8	11.12	98.6	43.8	48.7	3.92	54.3	9.8	126.0	86.5	92.4
1915	57.5	11.3	10.63	91.9	36.1	61.7	5.20	51.6	9.1	174.0	83.4	90.6
1916	88.9	19.6	11.22	160.3	52.4	146.1	5.69	88.1	14.7	248.6	122.1	88.9
1917	127.9	27.7	17.09	200.8	66.6	122.8	6.72	113.7	24.0	296.6	166.0	189.6
1918	136.5	27.6	20.13	204.2	70.9	119.3	7.28	91.7	28.0	340.1	151.6	191.8
1919	134.7	35.6	19.55	215.1	71.5	160.6	7.28	121.0	39.0	438.3	134.5	266.8
1920	67.7	14.0	16.72	144.3	47.2	116.4	5.77	70.7	21.1	176.6	127.8	118.9
1909–13												
Average value †	1563.1	777.2 ‡	757.1	599.9	424.7	215.1	156.6	108.5	101.9	31.6	25.2	19.6
Percentage distribution	32.8	16.2	15.8	12.5	8.9	4.5	3.3	2.3	2.1	0.7	0.5	0.4
1879–1920												
Range of values	24–36	11–18	13–25	11–25	6–11	3.5–6.5	1.0–4.5	1.2–2.9	1.0–4.5	0.2–1.0	0.5–1.2	0.2–0.9
Mode of values §	30	14	18	14	9	4	2.5	1.9	2.0	.6	.6	.2

* Most of the prices shown in the table were taken from the *Yearbook* of the Department of Agriculture or, for the last two years, from the *Monthly Crop Reporter*. The only exceptions to this statement occur in the cases of sugar, flaxseed, rice, and tobacco. Farm prices of tobacco, 1880–95, were taken from Circular 33 of the Department, *Tobacco Crop of the United States, 1612–1911*. For rice and flaxseed from 1880–1900 inclusive, and for sugar during the whole period covered, no farm prices were obtainable. In these instances wholesale market prices approximately on December 1 of each year were transcribed from the Aldrich *Report on Wholesale Prices, Wages, and Transportation in the United States* and from the *Bulletins* of the Bureau of Labor Statistics. These market prices for tobacco and cleaned rice were adjusted to the farm prices of subsequent years. No adjustment being possible for sugar, this series alone of the twelve, represents market prices of the raw product rather than farm prices.

† In millions of dollars.

‡ This figure does not include the value of cottonseed amounting to 142.7 million dollars. The weight originally used for cotton by Professor Day included cottonseed (appropriate in an index of agricultural production) and, therefore, was somewhat greater than those used in computing Fisher's indices. In the present study Day's indices have been recomputed with the value of cottonseed excluded so that the weight for cotton has been reduced from 18.7 (used by Day in the original computations) to 15.8 in the revised computation.

§ The distribution about the mode is, roughly, normal.

limit to differences that may be expected from the two methods of averaging and weighting actual data.

Having selected, first, the formulas to be tested with respect to each other (Fisher's index and the geometric average with constant weights) and second, the data, there remained a third question to be settled before the computations of the index numbers could proceed, i. e., the selection of a base year or period. We may use either a fixed period, say 1910, as the base for comparisons, or a variable period, say one in which each year is compared with the preceding year.[8] In computing the indices for the quinquennial years 1880 to 1920 we will use 1910 as the base year; in computing the annual indices, 1910–19, we will compute two indices for each year on the different bases: first, an index with the year 1910 as a base and second, an index with the preceding year as a moving base. The link relatives secured on a moving base are then converted into a chain series with 1910 as 100.

THE QUINQUENNIAL INDICES, 1880–1920: BASE, 1910

Indices of quantities produced and of the prices of twelve leading crops computed by Fisher's method of averaging and by taking the geometric average with constant weights are given in Table II. In the first column of that table is given the harmonic mean of the quantity relatives for twelve crops weighted by their respective values in the given year; and in the second column is given the arithmetic mean of the quantity relatives of the same twelve crops weighted by their values in the base year. In the third column is given

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[8] Irving Fisher favors the use of "chain index numbers" (Purchasing Power of Money, p. 204). The committee of the *British Association for the Advancement of Science*, appointed "for the purpose of investigating the best methods of ascertaining and measuring variations in the value of the monetary standard," recommended the use of the year preceding the given year as the base (*Report of the British Association for the Advancement of Science for 1887*, pp. 250, 252; other reports were published in 1888 and 1889). C. M. Walsh has endorsed this recommendation (*The Problem of Estimation*, pp. 105, 137).

TABLE II. — INDICES OF QUANTITIES PRODUCED AND OF PRICES OF TWELVE LEADING CROPS COMPUTED, FIRST, BY FISHER'S METHOD AND SECOND, BY TAKING THE GEOMETRIC AVERAGE OF THE RELATIVES WITH CONSTANT WEIGHTS: QUINQUENNIALLY, 1880–1920

(Base year, 1910)

Year	Indices of Quantities Produced				Corresponding Indices of Prices				Differences	
	Fisher's Method			Geometric average with constant weights † Q'_n	Fisher's Method			Geometric average with constant weights P'_n	Q_n less Q'_n	P_n less P'_n
	$\frac{\Sigma p_n q_n}{\Sigma p_n q_o}$ or F_1	$\frac{\Sigma p_o q_n}{\Sigma p_o q_o}$ or F_2	Q_n ("ideal") or $\sqrt{F_1 F_2}$		$\frac{\Sigma p_n q_n}{\Sigma p_o q_n}$	$\frac{\Sigma p_n q_o}{\Sigma p_o q_o}$	P_n ("ideal")			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1880	51.7	53.5	52.6	51.4	89.8	92.7	91.2	90.5	+1.2	+0.7
1885	57.1	58.9	58.0	57.8	73.2	75.4	74.3	74.1	+0.2	+0.2
1890	55.2	58.2	56.7	55.9	87.0	91.8	89.4	90.2	+0.8	−0.8
1895	70.1	72.2	71.1	71.2	58.2	59.9	59.0	58.3	−0.1	+0.7
1900	81.3	82.4	81.8	81.5	73.1	74.1	73.6	73.6	+0.3	0
1905	95.6	96.8	96.2	95.5	80.6	81.6	81.1	81.5	+0.7	−0.4
1910	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	0	0
1915	116.7	116.7	116.7	115.0	102.3	102.3	102.3	102.1	+1.7	+0.2
1920 *	121.6	120.6	121.1	119.9	140.9	139.8	140.3	140.7	+1.2	−0.4

* Sugar not included in 1920. † Day's index with weights revised to exclude cottonseed.

Fisher's index of quantities, Q_n (the geometric mean of the quantity relatives in the two preceding columns). The geometric mean of quantities with constant weights (Day's revised index of the physical volume of production for agriculture) is given in the fourth column for comparison with Fisher's index. Indices of prices corresponding to the indices of quantities just described are given in columns 5 to 8 of the table.

For the quinquennial years 1880–1920 Fisher's indices of quantities (Table II, column 3) relative to 1910 differ from Day's revised indices (the geometric averages with constant weights, column 4) for corresponding years by amounts ranging from −0.1 in 1895 to +1.7 in 1915. The differences between the two indices (column 9) are +1.2, +0.2, +0.8, −0.1, +0.3, +0.7, 0, +1.7 and +1.2 for the years 1880 to 1920, respectively. The maximum differences relative to the index numbers occurred in 1880 and amounted to 2.3 per cent. A graphic comparison of (1) the pair of ratios (F_1 and F_2) which Fisher averages to secure his index of quantities and (2) Day's index (Q'_n) is given in Chart I; Q_n is not plotted, but if plotted it would appear between F_1 and F_2. It is evident from this comparison that the two indices of quantities (Q_n and Q'_n) do not diverge in proportion to their distances from the base year but follow rather the same *general course* for the period 1880–1920.

Now let us turn our attention to the indices of prices. The divergence between Fisher's indices and the corre-

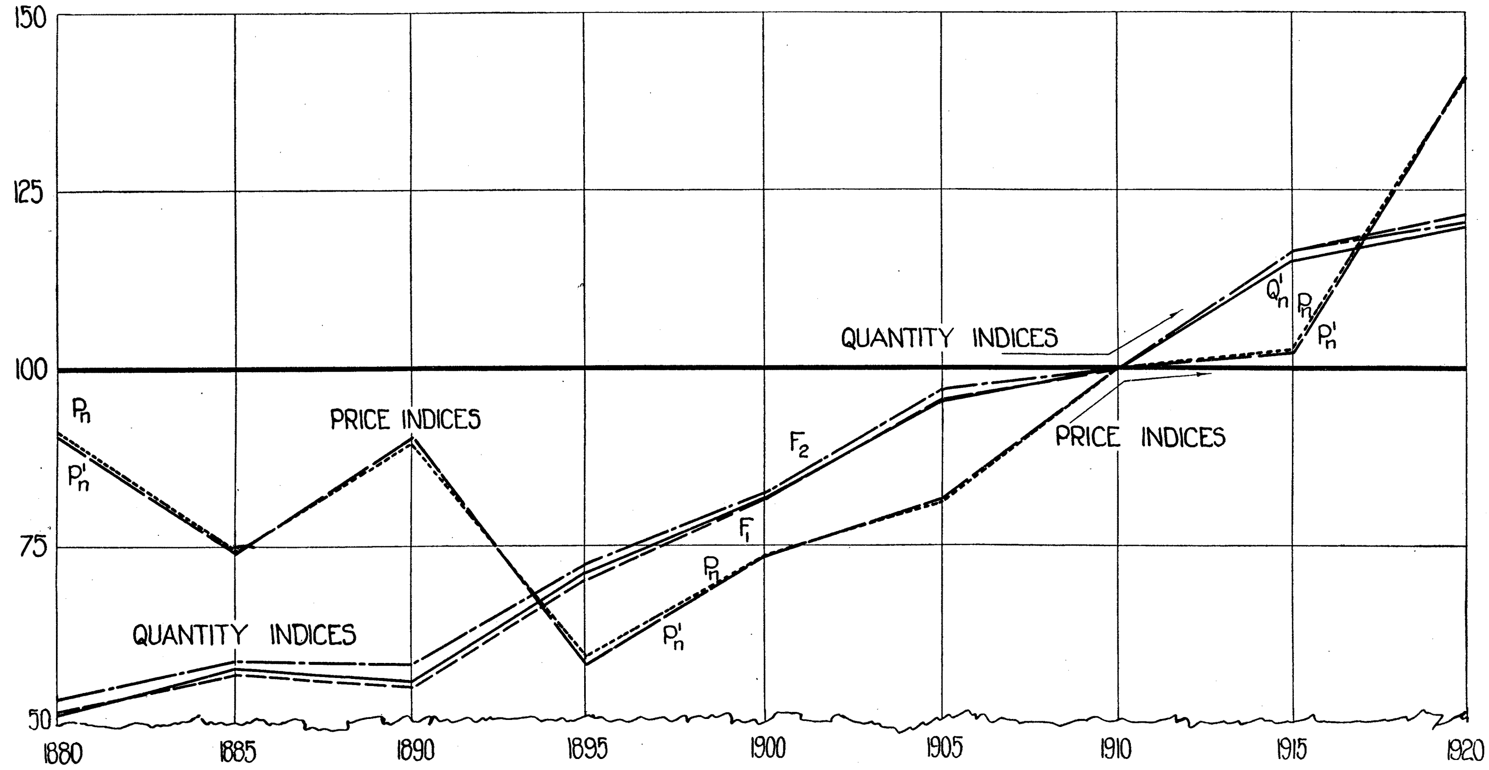

CHART I. — COMPARISON OF THE RATIOS (F_1 AND F_2) WHICH FISHER AVERAGES TO SECURE HIS INDEX OF QUANTITIES (Q_n) WITH DAY'S INDEX (Q'_n); ALSO COMPARISON OF FISHER'S PRICE INDEX (P_n) WITH THE GEOMETRIC AVERAGE HAVING CONSTANT WEIGHTS (P'_n).

sponding geometric averages with constant weights (columns 7, 8, and 10 of Table II) ranges from −0.8 in 1890 to +0.7 in 1880 and 1895. The differences between the two index numbers are +0.7, +0.2, −0.8, +0.7, 0, −0.4, 0, +0.2, and −0.4, respectively. The maximum relative difference, occurring in 1895, is about 1.2 per cent. A graphic comparison of the two price indices constructed according to the methods of Fisher and Day (P_n and P'_n) is given in Chart I.

The results from this comparison of the indices obtained by the two methods of averaging the data may be stated as follows: Indices of quantity or of prices of agricultural products of the United States for the period 1880–1920, when measured relative to a fixed base (1910 in this case), show the same general movement whether the Fisher method or the geometric average is used. During a period in which both production and prices more than doubled the maximum divergence of the paired index numbers was about 2 per cent for quantities and 1 per cent for prices. Furthermore, no cumulative divergence of the two indices is evident.

Are such differences as do occur between the two indices (reaching 2 per cent in one instance) significant? A direct answer cannot be given unless the question is made more specific. A divergence of 2 per cent is not significant when we desire merely to describe the course of production or of prices for the whole period; but such a divergence may be significant when we are concentrating our attention on changes that take place from one year to the next. For *some purposes*, certainly, a choice between the indices — the selection of one and the discarding of the other — appears to be indicated. It is hardly unreasonable to demand that, *given the data*, changes in the physical volume of production and prices be measured within 1 per cent.

Since our attention is now concentrated upon discrepancies of, say, 2 per cent or less and since such discrepancies are significant in a year-to-year comparison rather than in a 40 or 50-year survey, let us compare the paired indices annually for the period 1910–19.

THE ANNUAL INDICES, 1910–19: BASE, 1910

Table III presents Fisher's indices of quantities and prices (Q_n and P_n) and the geometric averages with constant weights (Q'_n and P'_n) for twelve leading crops, annually 1910–19. Both indices (Fisher's and the geometric average) are relative to the figures of the fixed base year, 1910. Columns 9 and 10 of the table give the differences between the pairs of indices for quantities and for prices, respectively. A graphic representation of the two indices of quantity (Q_n and Q'_n) is given in Chart III. The differences between the quantity indices amount to less than one-half of 1 per cent in seven years and to about 1.5 per cent in three years. The largest differences in the quantity indices (+1.7) occur in 1915 and 1919. A graphic representation of the two price indices (P_n and P'_n) is given in Chart II. The only difference in the price indices amounting to more than 1 per cent occurs in 1914. An attempt will be made to ascertain the exact reasons for the discrepancies between the paired indices for quantities in 1915 and 1919, and for prices in 1914.

First, consider the price indices for 1914. These indices are relative to the year 1910. Of the twelve crops entering the index four only have a weight of 10 per cent or more, i. e., corn, cotton, hay, and wheat (Table I). These four crops have a total weight of 77 per cent in Day's index. Therefore, the larger discrepancies between the two indices probably arise from some peculiarity of the data for one or more of these

TABLE III. — INDICES OF QUANTITIES PRODUCED AND OF PRICES OF TWELVE LEADING CROPS COMPUTED, FIRST, BY FISHER'S METHOD AND SECOND, BY TAKING THE GEOMETRIC AVERAGE OF THE RELATIVES WITH CONSTANT WEIGHTS: ANNUALLY, 1910–19

(*Base year, 1910*)

Year	INDICES OF QUANTITIES PRODUCED				CORRESPONDING INDICES OF PRICES				DIFFERENCES	
	Fisher's Method			Geometric average with constant weights * Q'_n	Fisher's Method			Geometric average with constant weights P'_n	Q_n less Q'_n	P_n less P'_n
	$\frac{\Sigma p_n q_n}{\Sigma p_n q_o}$	$\frac{\Sigma p_o q_n}{\Sigma p_o q_o}$	Q_n ("ideal")		$\frac{\Sigma p_n q_n}{\Sigma p_o q_n}$	$\frac{\Sigma p_n q_o}{\Sigma p_o q_o}$	P_n ("ideal")			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1910	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	0	0
1911	92.1	96.2	94.1	94.0	106.8	111.6	109.2	109.2	+0.1	0
1912	111.3	112.2	111.7	111.8	93.7	94.4	94.1	94.1	−0.1	0
1913	97.3	100.0	98.6	98.6	109.4	112.4	110.9	111.1	0	−0.2
1914	106.7	110.9	108.8	108.8	98.9	102.8	100.8	99.2	0	+1.6
1915	116.7	116.7	116.7	115.0	102.3	102.3	102.3	102.1	+1.7	+0.2
1916	100.0	103.6	101.8	101.4	151.3	156 8	154.0	154.2	+0.4	−0.2
1917	109.9	110.7	110.3	109.9	210.9	212.5	211.7	210.6	+0.4	+1.1
1918	108.9	110.8	109.8	108.2	220.3	224.2	222.2	222.3	+1.6	−0.1
1919	111.5	112.9	112.2	110.5	238.2	241.3	239.5	238.2	+1.7	+1.3

* Day's index with weights revised to exclude cottonseed.

four crops. Examining the price data of Table I for the four leading crops in the years 1910 and 1914 we find the most striking feature of the data to be the high price of cotton in 1910 (14.1 cents) and the low price in 1914 (6.8 cents). In these years the cotton crop was relatively low in 1910 (11.61 million bales) and high in 1914 (16.14 million bales. Between 1910 and 1914 the movement of the price of cotton was out of line with that of the other leading crops, and likewise the production of cotton was out of line with that of other crops. In 1914 occurred the largest cotton crop on record and the lowest December price since 1898. Because of the relatively heavy weight of cotton in the index numbers and because of the unusual figures for both prices and production in 1914 it seems probable that the discrepancy noted between Fisher's index and the geometric average in that year results from the different weights given to cotton in the two computations. In fact, if we omit cotton and compute Fisher's price index for the remaining eleven crops for 1914 we get 113.9 compared with the weighted geometric mean of 114.0. The difference between the paired price indices for eleven crops in 1914 is 0.1, compared with a difference of 1.6 between the indices for twelve crops. It is evident, therefore, that the discrepancy between the two price indices for twelve crops results from the different weights given in the two indices to the extremely low price relatives for cotton. A summary of the figures for 1914 follows:

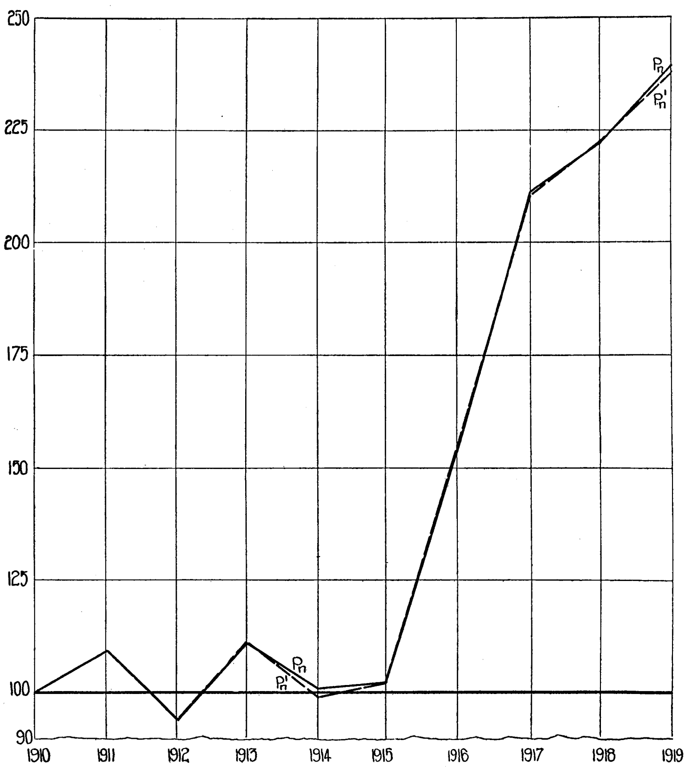

Chart II. — Comparison of Fisher's Price Index (P_n) with the Geometric Average having Constant Weights (P'_n).

(*Base year, 1910*)

	Fisher's method	Geometric average
Relative price of cotton	48.2	48.2
Average price of 11 crops	113.9	114.0
Average price of 12 crops	100.8	99.2

Now let us turn to the discrepancies between the paired indices of quantity in 1915 and 1919. Here again cotton is out of line with the other leading crops. The production of corn, wheat, and hay were all greater in 1915 and 1919 than in 1910 while the cotton crop was less. Fisher's quantity index for eleven crops (omitting cotton) in 1915 is 120.5 compared with the geometric average of 120.8; Fisher's index for these crops in 1919 is 115.2 compared with the geometric average of 114.8. The result is similar to that secured for prices, i. e., the elimination of cotton brings the indices closer together. A summary of the figures for 1915 and 1919 follows:

(*Base year, 1910*)

	1915		1919	
	Fisher's method	Geometric average	Fisher's method	Geometric average
Relative quantity of cotton..	96.4	96.4	98.0	98.0
Average quantity of 11 crops	120.5	120.8	115.2	114.8
Average quantity of 12 crops	116.7	115.0	112.2	110.5

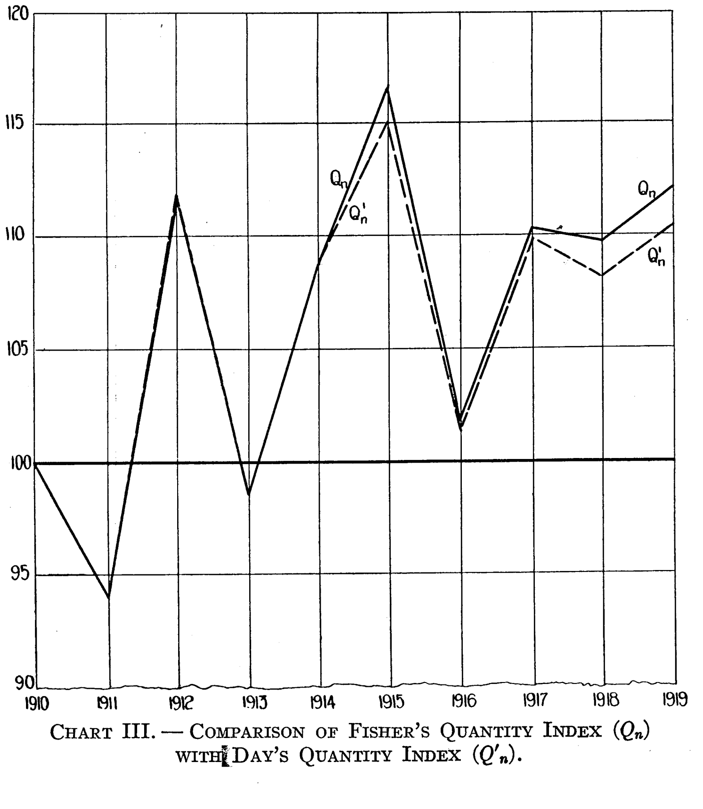

Chart III. — Comparison of Fisher's Quantity Index (Q_n) with Day's Quantity Index (Q'_n).

THE ANNUAL INDICES 1910–19: CHAIN RELATIVES

In the computations that have been made up to this point a fixed base year, 1910, has been used. According to the theory at the basis of Fisher's index an *exact* comparison of the prices or quantities of any two years can be made only if the prices and quantities for the two years in question *and those prices and quantities alone* are used in the computation of the indices; according to his theory constant or "normal" weights are not admissible. An *exact* comparison by Fisher's method of prices or quantities of a selected pair of years, say, 1919 and 1913 or 1919 and 1918, makes necessary the computation from the original data of two new indices having 1913 and 1918, respectively, as the bases. If, on the other hand, we use the geometric average with *constant weights*, the base may be shifted without error by the simple process of division. Thus, the geometric average price of 1919 relative to 1918 is 238.2 ÷ 222.3 = 107.2 (Table III).

In his paper on "The Best Form of Index Number," already referred to, Professor Fisher did not discuss the question of selection of a base. In other places, however, he has recommended the use of "chain" indices, that is, the computation of the index for each year with reference to the preceding year as the base and then "chaining" the links thus secured to secure a series of indices with reference to a fixed base. It is clear that as a matter of expediency in computing index numbers it is necessary either to select a fixed base or make use of the chain method. Otherwise it would be necessary in using Fisher's method (if we desire as he says "to make exact comparisons") to compute an index number for every pair of years that we wish to compare. If Professor Fisher could show that chaining his indices together results in very small errors and, further, that the errors are not cumulative he would meet, for practical purposes, the question here raised, i. e., is the error introduced by shifting the base an important one? In his paper Professor Fisher said, "for all practical purposes, the 'ideal' index number does fulfil Professor Persons' requirement (an index number on a fixed base should be usable for exact comparisons between two given years), as the deviation is usually less than half of 1 per cent even under the conditions of wide dispersement of prices and quantities in the war period of 1914–18."[9] In order to test the effect of shifting bases upon Fisher's index number we will compare his chain indices of prices and quantities, based upon the data already utilized, with his indices computed directly relative to the base year, 1910.

Table IV gives the link indices of quantities produced and of prices of twelve leading crops computed by Fisher's method. Columns 3 and 7 of that table give the relative production and price, respectively, each year compared with the preceding year. In each index, thus computed, only the prices and quantities of the various crops for the two adjacent years are utilized. In columns 4 and 8 are presented, for comparison with Fisher's link indices, the geometric averages relative to the preceding year. The biggest discrepancy between the pairs of link indices (Fisher's and the geometric average) occurs in 1915 and amounts to about 3 per cent. This discrepancy, as we have seen, results from a combination of the violent changes in cotton prices and production and the difference in weighting cotton in computing the paired indices. It is not the discrepancies between the two types of link relatives, however, that we are now interested in but rather the discrepancies presented in columns 9 and 10 of Table V.

The indices given in Table V are all computed by Fisher's method. Column 3 of that table is obtained from column 3 of Table IV by a process of progressive multiplication. Thus the quantity chain index for 1912 is 94.1 × 119.2 = 112.1, and the index for 1913 is 112.1 × 88.1 = 98.6. In this way the indices in columns 3 and 7 are shifted to the base 1910. In columns 4 and 8 are given Fisher's indices computed *directly* to the base 1910 (see columns 3 and 7 of Table III). The last two columns of Table V give the differences between, first, Fisher's chain indices for quantities and prices and, second, his indices computed directly to the base year

TABLE IV. — LINK INDICES OF QUANTITIES PRODUCED AND OF PRICES OF TWELVE LEADING CROPS COMPUTED, FIRST, BY FISHER'S METHOD AND SECOND, BY TAKING THE GEOMETRIC AVERAGE OF THE RELATIVES WITH CONSTANT WEIGHTS: ANNUALLY 1910–19

(*The preceding year is the base*)

Year	LINK INDICES OF QUANTITIES				CORRESPONDING LINK INDICES OF PRICES				DIFFERENCES	
	Fisher's Method *			Geometric average with constant weights Q'_n (link) †	Fisher's Method *			Geometric average with constant weights P'_n (link) †	Column 3 less column 4	Column 7 less column 8
	$\frac{\Sigma p_n q_n}{\Sigma p_n q_{n-1}}$	$\frac{\Sigma p_{n-1} q_n}{\Sigma p_{n-1} q_{n-1}}$	Q_n (link)		$\frac{\Sigma p_n q_n}{\Sigma p_{n-1} q_n}$	$\frac{\Sigma p_n q_{n-1}}{\Sigma p_{n-1} q_{n-1}}$	P_n (link)			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1910	103.0	103.7	103.4	104.0	93.3	94.0	93.6	93.2	−0.6	+0.4
1911	92.1	96.2	94.1	94.0	106.8	111.6	109.2	109.2	+0.1	0
1912	117.2	121.2	119.2	118.9	84.4	87.3	85.9	86.2	+0.3	−0.3
1913	87.5	88.7	88.1	88.2	117.4	119.0	118.2	118.1	−0.1	+0.1
1914	110.5	110.5	110.5	110.3	90.8	90.8	90.8	89.2	+0.2	+1.6
1915	106.9	110.7	108.8	105.7	98.4	101.9	100.1	103.0	+3.1	−2.9
1916	86.0	87.8	86.9	88.2	149.4	152.6	151.0	151.1	−1.3	−0.1
1917	109.2	110.3	109.7	108.3	135.0	136.4	135.7	136.5	+1.4	−0.8
1918	99.0	99.5	99.2	98.5	105.1	105.6	105.3	105.6	+0.7	−0.3
1919	101.8	106.8	104.2	102.0	103.2	108.3	105.7	107.2	+2.2	−1.5

* Computed to hundredths of a per cent and rounded to nearest tenth.

† Computed by taking the ratios between consecutive items of columns 4 and 8 of Table III.

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[9] *Quarterly Publication of the American Statistical Association*, March, 1921, p. 549.

TABLE V. — INDICES OF QUANTITIES PRODUCED AND OF PRICES OF TWELVE LEADING CROPS COMPUTED BY FISHER'S METHOD, FIRST, USING THE PRECEDING YEAR AS THE BASE AND THEN ADJUSTING TO 1910 (CHAIN INDICES) AND SECOND, TAKING 1910 DIRECTLY AS THE BASE YEAR

(*Base year, 1910*)

Year	Indices of Quantities Produced				Corresponding Indices of Prices				Differences	
	Chain Indices *			Q_n ("ideal")	Chain Indices *			P_n ("ideal")	Q_n ("ideal") less Q_n (chain)	P_n ("ideal") less P_n (chain)
	A	*B*	Q_n (chain) i.e. $\sqrt{A \cdot B}$		*C*	*D*	P_n (chain) i.e. $\sqrt{C \cdot D}$			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1910	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	0	0
1911	92.1	96.2	94.1	94.1	106.8	111.6	109.2	109.2	0	0
1912	107.9	116.6	112.1	111.7	90.2	97.4	93.7	94.1	−0.4	+0.4
1913	94.4	103.4	98.8	98.6	105.9	115.9	110.8	110.9	−0.2	+0.1
1914	104.3	114.2	109.1	108.8	96.1	105.2	100.6	100.8	−0.3	+0.2
1915	111.4	126.4	118.7	116.7	94.5	107.2	100.7	102.3	−2.0	+1.6
1916	95.8	111.0	103.1	101.8	141.2	163.6	152.0	154.0	−1.3	+2.0
1917	104.6	122.4	113.1	110.3	190.7	223.3	206.4	211.7	−2.8	+5.3
1918	103.5	121.8	112.3	109.8	200.4	235.8	217.4	222.2	−2.5	+4.8
1919	105.3	130.1	117.0	112.2	206.7	255.3	229.7	239.5	−4.8	+9.8

* Progressive products of link indices taken to hundredths (see corresponding columns of Table IV).

1910. It will be seen that the differences between Fisher's two indices for quantities and for prices, respectively, are small until 1915. After that year the differences grow larger until in 1919 they reach 4 per cent for both pairs of indices (Chart IV). The statement made by Professor Fisher that "the deviation is usually less than half of 1 per cent even under the conditions of wide dispersement of prices and quantities in the war period of 1914–18" is not substantiated.[10] In the present test it appears that the use of chain relatives introduces a cumulative error which, for both quantity and price indices, reaches a maximum of 4 per cent in the ten-year period covered. In fact if we take Fisher's indices for 1910–19 computed directly to the base 1910 as the standard and compare with this standard, first, Fisher's chain indices, and second, the indices computed as geometric averages with constant weights, the indices last named conform closer to the standard than do the chain indices. The average divergence, disregarding algebraic signs, of the geometric indices for quantity from the standard is 0.6 compared with 1.4 for the chain indices; the average divergence of the geometric indices for prices from the standard is 0.5 compared with 2.4 for the chain indices. That is using Fisher's "ideal" index as a standard the geometric average with constant weights is better than Fisher's chain index.

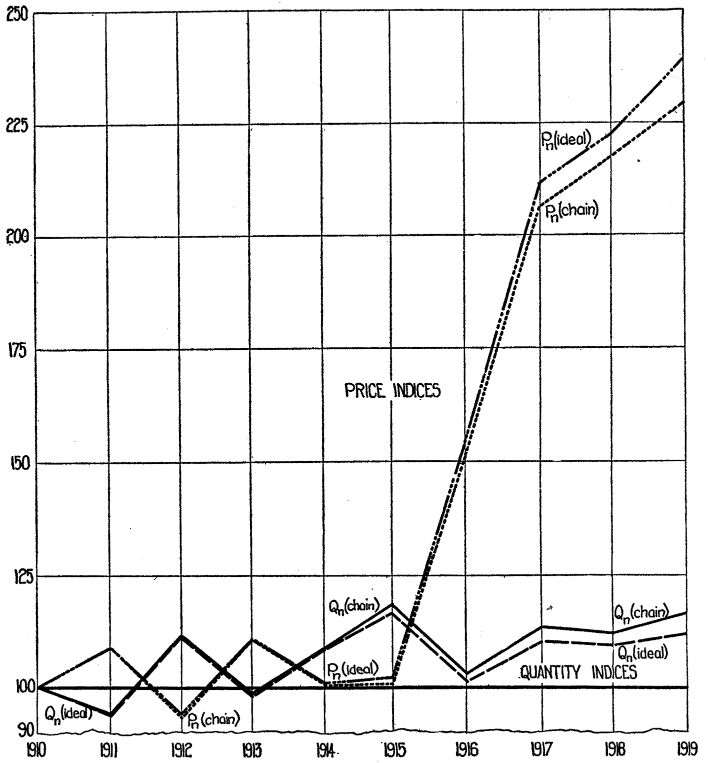

CHART IV. — COMPARISON OF FISHER'S "IDEAL" INDICES FOR PRICES (P_n) AND QUANTITIES (Q_n) WITH HIS CHAIN INDICES (P_n AND Q_n).

Another test that may be applied to Fisher's chain series is the "circular" test. Suppose that we assume a fictitious year 1920 *with precisely the same prices and quantities as those of the base year 1910.* Computing the link indices for this fictitious year relative to 1919 and completing the chain, we find that the indices of prices and quantities for the fictitious year are 95.8 and 104.3, instead of 100. That is, Fisher's chain indices fail to meet the circular test by more than 4 per cent while the geometric mean with constant weights satisfies the test exactly.[11]

THE EFFECTS OF DIFFERENCES IN WEIGHTING

Our investigations thus far point to the conclusion that the differences between Fisher's indices and the geometric averages with constant weights result primarily from differences in weighting. Fisher's formula for price indices is a function of two variables, prices, and weights, while our geometric average is another function of one variable, prices, and constant weights. Fisher's formula differs from the geometric average with constant weights, then, in two respects: first, the mathematical processes according to which the price relatives are combined; and, second, the weights of those relatives. We have found in the preceding study that even with these two points of difference the maximum divergence between the two sets of indices is only about 2 per cent.

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[10] This statement holds true for the indices which are given in Table V for the period 1910–14 only.

[11] Fisher's formulas for index numbers (both "ideal" and chain) satisfy the criterion $P_nQ_n = \frac{V_n}{V_o}$. That is, the product of Fisher's index number of prices in a given year and the correlative index number for quantities in that year will give the ratio of aggregate values in the given year to the aggregate

Fisher's ideal indices and chain indices are computed by applying precisely the same mathematical function or formula in both cases. To be sure, the process of chaining introduces a set of weights different from that of the ideal index. This different set of weights thus introduced, is responsible for a divergence amounting to 4 per cent between Fisher's ideal and chain indices.

It appears from the considerations named in the preceding paragraphs that the divergences between these three series of index numbers are due more to differences in weighting than to differences in the mathematical formulas applied. In order to test this conclusion we will compute a new geometric average of prices based, as are the others, on the data of 12 leading agricultural products but using a new set of weights. The new weights are obtained from the United States Bureau of Labor Statistics and are based upon the total value of each commodity marketed in the United States in 1909. The weights based upon values produced in 1909–13, which we have used in the preceding sections, and the weights obtained from the Bureau of Labor Statistics are presented in Table VI.

TABLE VI. — COMPARISON OF WEIGHTS OBTAINED FROM THE UNITED STATES BUREAU OF LABOR STATISTICS AND THOSE USED BY DAY IN HIS INDEX OF PRODUCTION FOR AGRICULTURE.

Crop	"Value marketed" in 1909 * (*in millions*)	Percentage distribution of values marketed in 1909	Percentage distribution of values produced 1909–13 †
Corn	$308	9.4	32.8
Cotton	655	20.0	16.2
Hay	144	4.4	15.8
Wheat	820	25.1	12.5
Oats	129	3.9	8.9
White potatoes	273	8.3	4.5
Sugar	603	18.4	3.3
Barley	51	1.6	2.3
Tobacco	194	5.9	2.1
Flaxseed	31	1.0	0.7
Rye	23	0.7	0.5
Rice	42	1.3	0.4
	3,273	100.0	100.0

* As given in the *Bulletin* of the United States Bureau of Labor Statistics, No. 269, pp. 182–185. † See Table I above, p. 105.

The new geometric averages (P''_n) of the prices of twelve leading crops weighted by value marketed in 1909 are given in Table VII, columns 2 and 5. The old geometric averages (P'_n) of the prices of twelve leading crops weighted by the average value of production in 1909–13 are given in the same table, columns 1 and 4, for comparison. The differences between the old and new indices (P'_n-P''_n) quinquennially, 1880–1920, and annually, 1910–19, are given in columns 3 and 6. A graphic representation of the pairs of indices is given in Chart V. The divergence between the two geometric averages, each with constant weights, reaches a maximum of −13.9, in the period 1880–1920 and of +11.3 in the period 1910–19. The average divergence, disregarding algebraic signs, for both periods is 5.2 points. The average divergence, disregarding signs, between the old geometric average of prices and Fisher's ideal index for the same periods is less than 0.5 and the maximum divergence is only +1.6 points. It appears from this comparison that the principle upon which weights are selected (value marketed or value produced) gives

TABLE VII. — GEOMETRIC AVERAGES OF THE RELATIVE PRICES OF TWELVE LEADING CROPS WEIGHTED, FIRST, BY VALUES PRODUCED IN 1909–13 AND, SECOND, BY VALUES MARKETED IN 1909.

(*Base year, 1910*)

Year	Geometric averages weighted by value of production in 1909–13 P'_n	Geometric averages weighted by value marketed in 1909 P''_n	P'_n less P''_n	Year	Geometric averages weighted by value of production in 1909–13 P'_n	Geometric averages weighted by value marketed in 1909 P''_n	P'_n less P''_n
	(1)	(2)	(3)		(4)	(5)	(6)
1880	90.5	104.4	−13.9	1910	100.0	100.0	0
1885	74.1	86.5	−12.4	1911	109.2	102.5	+ 6.7
1890	90.2	95.0	− 4.8	1912	94.1	93.0	+ 1.1
1895	58.3	62.0	− 3.7	1913	111.1	99.8	+11.3
1900	73.6	77.6	− 4.0	1914	99.2	91.0	+ 8.2
1905	81.5	85.1	− 3.6	1915	102.1	103.8	− 1.7
1910	100.0	100.0	0	1916	154.2	161.1	− 6.9
1915	102.1	103.8	− 1.7	1917	210.6	207.0	+ 3.6
1920*	140.7	145.3	− 4.6	1918	222.3	215.2	+ 7.1
				1919	238.2	241.6	− 3.4

* Sugar not included in 1920.

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values in the base year. The geometric average with constant weights will satisfy the criterion just stated *only approximately*. For the data and years tested in the present study the maximum divergence of the geometric average, $P'_nQ'_n$, from $\frac{V_n}{V_o}$ is about 3 per cent (in 1880). The figures are given in the following table:

(*Base year, 1910*)

Year	$\frac{V_n}{V_o}$	$P'_nQ'_n$	$\frac{V_n}{V_o} - P'_nQ'_n$	Year	$\frac{V_n}{V_o}$	$P'_nQ'_n$	$\frac{V_n}{V_o} - P'_nQ'_n$
1880	48.0	46.5	+1.5	1910	100.0	100.0	0
1885	43.1	42.8	+0.3	1911	102.7	102.6	+0.1
1890	50.6	50.4	+0.2	1912	105.1	105.2	−0.1
1895	42.0	41.5	+0.5	1913	109.4	109.5	−0.1
1900	60.2	60.0	+0.2	1914	109.7	107.9	+1.8
1905	78.0	77.8	+0.2	1915	119.5	117.4	+2.1
1910	100.0	100.0	0	1916	156.8	156.4	+0.4
1915	119.5	117.4	+2.1	1917	233.5	231.4	+2.1
1920	169.9	168.7	+1.2	1918	244.1	240.5	+3.6
	....	....	....	1919	269.0	263.2	+5.8

rise to much greater differences than does the principle of combining the relatives, e. g., the selection of the geometric instead of Fisher's formula for averaging.

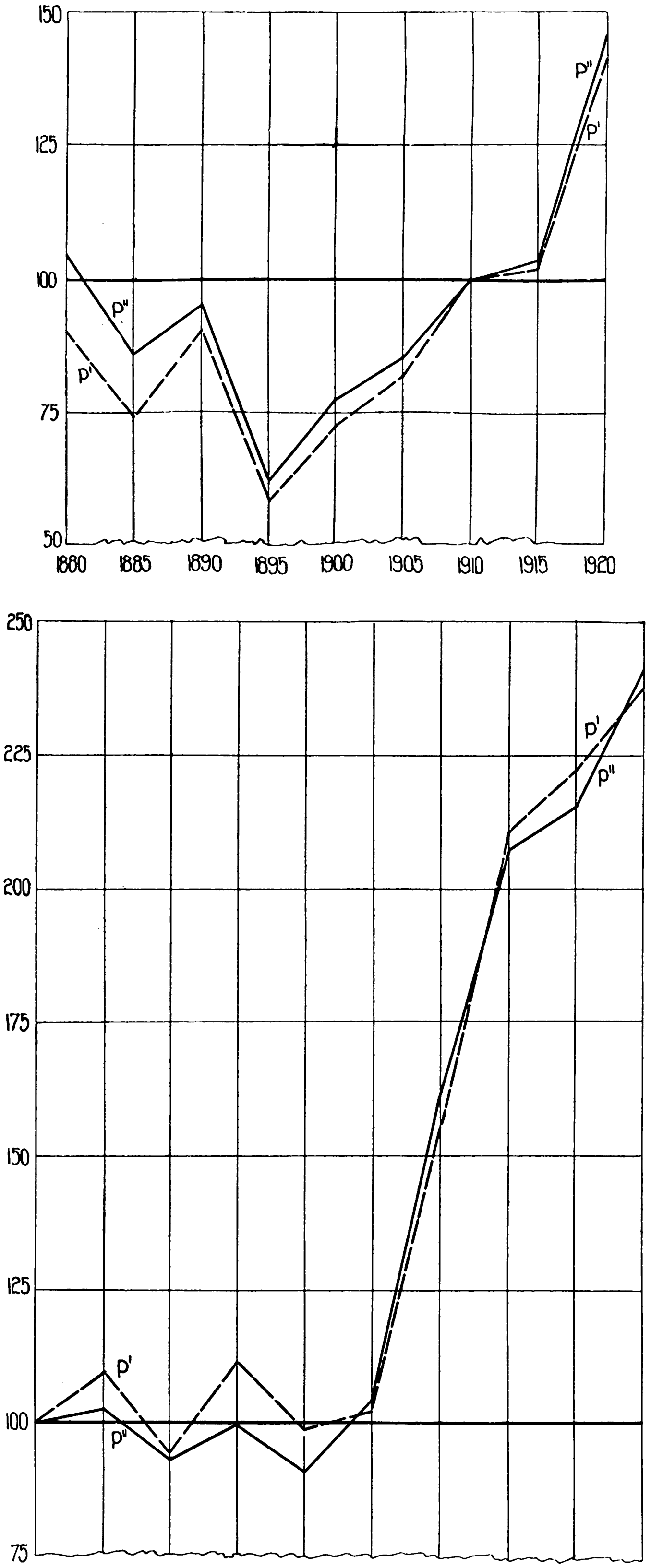

Chart V. — Comparison of Geometric Averages of the Relative Prices of Twelve Leading Crops Weighted, First, by Values Produced in 1909–13 (P'_n) and Second, by Values Marketed in 1909 (P''_n).

SUMMARY AND CONCLUSION

1. Fisher's index and the geometric average with constant weights, both computed from the quantity and price data for twelve leading crops and with the fixed base year 1910, show the same *general course* of production and prices, quinquennially 1880–1920, and annually 1910–19.

2. The maximum discrepancy between the two types of quantity indices was 2.3 per cent; and the maximum discrepancy between the two types of price indices was 1.6 per cent (Tables II and III). These discrepancies are due to differences in weighting, especially to differences in weighting commodities like cotton, having violent fluctuations in prices or production.

3. Chain indices computed by Fisher's method tend to diverge from his indices computed directly to a fixed base. For the period and data tested this divergence was cumulative and reached 4 per cent for both quantity and price indices. For the period of violent price changes from 1910 to 1919 the geometric average with constant weights is consistently much closer to Fisher's index on the fixed base 1910 than is Fisher's chain index.

4. Fisher's formula for index numbers is an excellent one if we desire to compare average prices or average quantities of any two years *using as our weights not constant or "normal" values but values of the two years compared;* indices for a series of years should be computed to a fixed base if we are to avoid the inaccuracies introduced by the process of shifting bases.

Finally, it may be added that the computations made in connection with the present article indicate, above all, that weighting is exceedingly important in the construction of index numbers. If the construction of an index number be undertaken one of the most important questions to be settled is that of the logical weights to use; logical, that is, with respect to the purpose of the proposed index. Professor Fisher apparently holds that the same p's and q's should be used for both indices of

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[12] Fisher's index measures neither the varying cost of a constant amount of goods nor the varying amount of goods which a dollar will buy.

[13] If constant weights are used in Fisher's formula that formula becomes the ratio between two aggregates.

The object of the present article is, as we have said, to compare Fisher's indices with geometric averages having constant weights. Other averages have also been computed from the same data, however, to compare with the geometric average. The following table gives index numbers of physical production for agriculture computed by taking the ratios of (*a*) the aggregate values of twelve leading crops in each year *at average prices for 1909–13* to (*b*) the aggregate values of these crops in 1910 also *at average prices for 1909–13*:

(*Base year, 1910*)

Year	$\frac{\Sigma q_n p_{1909-13}}{\Sigma q_{1910} p_{1909-13}}$	Divergence from Q'_n	Year	$\frac{\Sigma q_n p_{1909-13}}{\Sigma q_{1910} p_{1909-13}}$	Divergence from Q'_n
	(1)	(2)		(1)	(2)
1880	53.2	+1.8	1910	100.0	0
1885	59.1	+1.3	1911	94.4	+0.4
1890	57.1	+1.2	1912	111.7	−0.1
1895	72.6	+1.4	1913	98.3	−0.3
1900	82.0	+0.5	1914	109.0	+ .2
1905	96.2	+0.7	1915	116.3	+1.3
1910	100.0	0	1916	102.9	+1.5
1915	116.3	+1.3	1917	111.2	+1.3
1920	120.4 *	+0.5	1918	109.9	+1.7
			1919	112.4	+1.9

* Sugar omitted.

It will be noticed from the above table that the divergence between the ratios of aggregates (column 1) and the geometric average with constant weights (Tables II and III) was greater than 2 per cent in three years only, 1880, 1885, and 1890.

prices and indices of production. I contend that the quantities chosen (whether total production, the portion of that production which enters the market, the number of units exchanged, or some other figure) for a price index depend upon the purpose for which the index is constructed. Because one set of quantities is called for in our price index does not mean that the *same* set of quantities gives a useful quantity index. Perhaps the most widely useful price index is one weighted by quantities exchanged, and the most useful quantity index is one based upon quantities produced. In other words, Fisher's correlative indices of prices and quantities do not correspond to the useful concepts of "prices of goods exchanged" and "quantities of goods produced."

The type of average, as well as the weights, selected for index numbers depends upon the purpose of the index numbers. Fisher's formula is "the best" measure of general exchange value[12] if we desire to compare two years only, and not a series of years each with the other; his correlative formulas for prices and quantities are "the best" only if we are interested in the *same quantities* in both cases. If we compute not yearly but monthly index numbers by Fisher's method it is obvious that the lack of monthly data for quantities and the burden of computation would necessitate a compromise, and constant rather than variable weights would have to be used.[13] The geometric average with constant weights (or weights changed only occasionally) appears to be "ideally" better than Fisher's formula if we desire to compare a series of years each with the other. In addition, the geometric average has the practical advantage that fewer data and half the labor are required for its computation. The accuracy of the geometric mean combined with relative ease in calculation make it superior for most purposes to Fisher's index.